

1. A block of mass $m_A = 2$ kg is resting on the horizontal top surface of block B of mass $m_B = 3$ kg. Block B is on a flat surface, the masses are tied to each other with a horizontal ideal string that is passing over a frictionless, massless pulley as shown in the figure. Both the surface between the blocks, and the surface between the block and the floor have a friction coefficient $\mu = 0.1$, the static and kinetic friction coefficients can be taken as equal. Initially, the left side of the upper block is 50 cm to the right of the lower block. Gravitational acceleration is $g = 10$ m/s².

A horizontal force F acts to the left on the upper block.

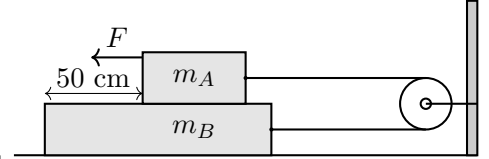
- [5 Pts.] (a) What is the minimum F so that the system starts to move?

If the magnitude of the force is $F = 20$ N:

- [5 Pts.] (b) What are the accelerations of the masses?

- [5 Pts.] (c) Calculate the total work done by the external force F until the left sides of the blocks are aligned.

- [10 Pts.] (d) Find the total energy lost to heat until the left sides of the blocks are aligned.



a)

$$F_{NA} = m_A g, \quad f_1 \leq \mu F_{NA}$$

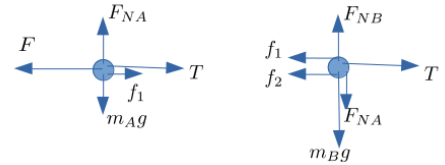
$$F = T + f_1, \quad f_2 \leq \mu F_{NB}$$

$$F_{NB} = F_{NA} + m_B g$$

$$T = f_1 + f_2 \Rightarrow F = 2f_1 + f_2$$

So, $F \leq 2\mu m_A g + \mu(m_A + m_B)g$ for static situation $\Rightarrow$ motion starts if $F > (3m_A + m_B)g\mu$

$$F_{min} = 9 \text{ N}$$



b)

$$F - T - f_1 = m_A a$$

$$T - f_1 - f_2 = m_B a \quad \Rightarrow a = \frac{F - (3m_A + m_B)g\mu}{(m_A + m_B)} = \frac{20 - 9}{5} = \frac{11}{5} = 2.2 \text{ m/s}^2$$

$$F - 2f_1 - f_2 = (m_A + m_B)a$$

c) m_A has to move $d = 25$ cm for alignment

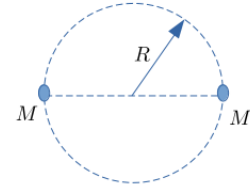
$$W_F = \mathbf{F} \cdot \mathbf{d} = 20 \times 0.25 = 5 \text{ J}$$

d)

$$U_f = df_1 + df_2 + df_1 = d(2f_1 + f_2) = 0.25 \times 9 = 2.25 \text{ J}$$

2. Two identical stars with mass M orbit around their common center of mass. Each orbit is circular and has radius R , so that the two stars are always on opposite sides of the circle.

- [5 Pts.] (a) Find the magnitude of gravitational force of one star on the other.
 [10 Pts.] (b) Find the orbital speed of each star and the period of the orbit.
 [5 Pts.] (c) What is the total mechanical energy?
 [5 Pts.] (d) How much energy would be required to separate the two stars to infinity?



a)

$$F = G \frac{MM}{(2R)^2} = G \frac{M^2}{4R^2}$$

b)

$$F = ma \quad \Rightarrow \quad G \frac{M^2}{4R^2} = \frac{Mv^2}{R}$$

Then,

$$v = \frac{1}{2} \sqrt{\frac{GM}{R}}$$

So, the period is

$$T = \frac{2\pi R}{v} = 4\pi \sqrt{\frac{R^3}{GM}}$$

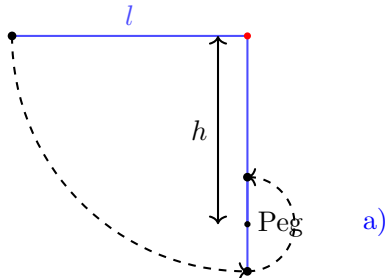
c)

$$E = -G \frac{MM}{2R} + 2 \times \frac{1}{2} Mv^2 = -G \frac{M^2}{4R}$$

d) $U = -E = G \frac{M^2}{4R}$ must be supplied

3. A ball is attached to a long horizontal cord l whose other end is fixed, as shown in the figure.

- [5 Pts.] (a) If the ball is released, what will be its speed at the lowest point of its path?
- [10 Pts.] (b) A peg is located a distance h directly below the point of attachment of the cord. If $h = 4l/5$, what will be the speed of the ball when it reaches the top of its circular path about the peg?
- [10 Pts.] (c) Find the minimum h for the ball to make a complete circle about the peg.



$$Mgl = \frac{1}{2}mv_B^2 \Rightarrow v_B = \sqrt{2gl}$$

Energy is conserved

b) Height difference is $\Delta y = h - (l - h) = 2h - l$

Energy conservation gives

$$Mg(2h - l) = \frac{1}{2}Mv_T^2$$

$$v_T = \sqrt{2g(2h - l)} = \sqrt{\frac{6}{5}gl}$$

c) We need

$$M \frac{v_T^2}{l - h} = Mg$$

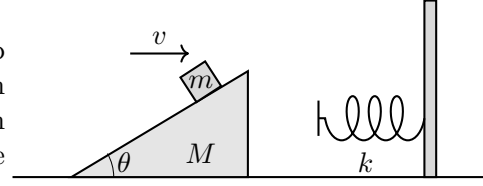
So,

$$2g(2h - l) = g(l - h)$$

$$4h - 2l = l - h \Rightarrow 5h = 3l$$

which means $h \geq \frac{3}{5}l$ and $h_{min} = \frac{3}{5}l$

4. A block of mass M is shaped like an inclined plane of angle θ . On top of this inclined block rests a smaller mass m . There is friction between the masses but the floor is frictionless. The friction coefficient between the blocks is high enough so that the small mass does not slide on the lower block.



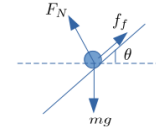
Initially the masses are moving with common velocity v towards an ideal spring tied to a wall. The spring constant is k and the gravitational acceleration is given as g .

- [5 Pts.] (a) Assuming that the upper mass does not slip during the motion, find the maximum compression of the spring.
- [5 Pts.] (b) Calculate the minimum static friction coefficient between the masses so that the small mass does not start sliding down when both blocks moving with constant speed before they hit the spring.
- [15 Pts.] (c) Calculate the minimum static friction coefficient between the masses so that the small mass does not slide on the inclined block at any time during the motion.

a) Using energy conservation

$$\frac{1}{2}(M+m)v^2 = \frac{1}{2}kx_{max}^2 \Rightarrow x_{max} = \sqrt{\frac{M+m}{k}}v$$

b) Inertial frame, all net forces must be zero

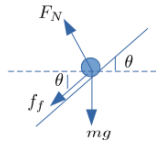


$$\begin{aligned} F_N \cos \theta + f_f \sin \theta &= mg \\ F_N \sin \theta &= f_f \cos \theta \Rightarrow f_f = F_N \tan \theta \leq \mu F_N \end{aligned}$$

We need

$$\mu \geq \tan \theta$$

c) If the force applied by the spring is over a threshold m would slide upward. The most critical moment is during the maximum compression of the spring.



$$\begin{aligned} a &= \frac{F_{max}}{m+M} = \frac{kx_{max}}{m+M} = \sqrt{\frac{k}{m+M}}v \\ F_N \sin \theta + f_f \cos \theta &= ma \Rightarrow F_N = m(a \sin \theta + g \cos \theta) \\ F_N \cos \theta - f_f \sin \theta &= mg \Rightarrow f_f = m(a \cos \theta - g \sin \theta) \end{aligned}$$

We need

$$f_f \leq \mu F_N$$

Then,

$$\mu_{min} = \frac{a \cos \theta - g \sin \theta}{a \sin \theta + g \cos \theta} = \frac{\sqrt{k/(M+m)}v \cos \theta - g \sin \theta}{\sqrt{k/(M+m)}v \sin \theta + g \cos \theta}$$